

Multirate Integration of a European Power System Network Model

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Abstract. An efficient approach for the numerical integration of a European power system network model is described. The time evolution of the power system is modeled by a system of differential and algebraic equations. In standard numerical time integration methods the most active variables impose the time step for the whole system. We describe an approach, which allows the use of different, local time steps over the variables. The partitioning of the components of the system in different classes of activity is based on the topology of the power system.

Keywords: multirate time stepping, local time stepping, differential and algebraic equations

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INTRODUCTION

Modeling of power systems results in large differential-algebraic systems (DAEs). These systems are built from the equations describing the network, the generators, the voltage regulators, the speed governors and the dynamic loads. For the numerical solution of systems of DAEs there are many methods available; see for example the text books of Butcher [1], Hairer et al. [2, 3], Lambert [4]. These methods use time steps that are varying in time, but are constant over the system variables.

Complex power networks are modeled by very large systems of which some components may exhibit a significantly more active behavior than others, distinguishing slow and rapid temporal variations. A voltage wave propagation due to lightning lasts a few microseconds to milliseconds but a secondary frequency control may have a time duration of several minutes. A particular situation requiring numerical simulation is that a damaging event which occurs in one of the European countries should not affect other countries. Such systems can be efficiently solved using multirate methods [5, 6, 7, 8]. Multirate methods attempt to take large time steps for slowly varying components and small steps for components with a significantly more rapid variation, so as to speed up the numerical computations.

MULTIRATE TIME STEPPING

Our multirate approach is based on local temporal error estimation. Given a global time step, we compute a first, tentative approximation at the new time level for all components. For those components for which the error estimator indicates that smaller steps are needed, the computation is redone with smaller time steps. During this refinement stage we perform a local variable time stepping. This refinement stage may require values at the intermediate time level of components that are not refined. These values can be obtained by interpolation. In this case we have two levels of activity: slow variables and fast variables.

We can also allow for more levels of activity. In this case, the desired accuracy does not necessarily have to be achieved during the first refinement. The refinement can be continued until the error estimator is below a prescribed tolerance for all components. Such approach was considered in [9]. For the sake of simplicity, in this paper we will assume that only one level of refinement is performed.

MIXED ADAMS-BDF TIME INTEGRATION METHOD

As the basic time integration method we use the mixed Adams-BDF method presented in [10]. The Adams method is applied to the differential state variables and provides a reliable detection of unstable situations. The BDF method

is used for the algebraic state variables, since it is less sensitive to the variations in the algebraic equations than the Adams method. Nordsieck formulation of the methods is used. Nordsieck methods are equivalent to multistep methods and allow a convenient way of changing the step size, which is a major element of multirate methods.

We consider a prototype ordinary differential equation (ODE) system

$$y' = f(t, y) \quad (1)$$

with initial solution $y(0) = y_0$ and denote by y_n the numerical approximation of the solution at time t_n and by τ the time step size. Multistep methods use the values of the solution and its derivative at the previous k steps in order to compute a high-order approximation of the solution for the next step. Nordsieck suggested storing the vector

$$z_n = \left(y_n, \tau y'_n, \frac{\tau^2}{2!} y''_n, \dots, \frac{\tau^k}{k!} y_n^{(k)} \right) \quad (2)$$

instead. In compact form the Nordsieck methods can be written as

$$z_{n+1} = (P \otimes I) z_n + (l \otimes I) (y_{n+1} - (e_1^T P \otimes I) z_n)^T. \quad (3)$$

Here P is the Pascal triangle matrix defined by

$$p_{ij} = \begin{cases} \binom{j}{i} & \text{for } 0 \leq i \leq j \leq k, \\ 0 & \text{otherwise,} \end{cases} \quad (4)$$

with $e_1 = (1, 0, 0, \dots, 0)^T$ and method coefficients $l = (l_0, l_1, \dots, l_k)^T$.

PARTITIONING STRATEGY

Partitioning of the variables in slow and fast can be fixed and given in advance, or it can vary in time and should be performed automatically during the time integration process.

In this section we present a strategy for automatic partitioning of the differential and algebraic variables. This strategy is based on the local time variation of the numerical solution of the system and on the topology of the power system.

A power system can be usually decomposed in two parts:

- a large network which consists of a set of nodes (each node introducing two variables) connected by a set of branches (lines, cables and transformers).
- a set of components (synchronous machines, motors, loads. . .) which are usually connected to a particular node.

This particular structure can be used to derive a dedicated partitioning strategy.

We first perform a single step with step size τ and using an error estimator we determine the variables which do not satisfy the criterion

$$e_i < Tol, \quad (5)$$

where e_i is the estimated local error for the variable i and Tol is a given tolerance. These variables will be called fast. To allow for accurate computation of the fast variables, during the refinement stage, we also recompute the slow variables which are strongly coupled to the fast ones.

The propagation of the fast status is performed as follows:

- all the components which contain at least one fast variable are classified as fast.
- all the nodes which contain at least one fast variable are classified as fast.
- the connection nodes of the fast components are classified as fast.
- the fast status of the nodes is then propagated through the network:
 - a graph G is defined as follows:
 - * a node in G is associated to each electrical node.

- * an edge is defined between two nodes of G if there exists at least one branch linking the two corresponding electrical nodes.
- * a weight representing the “electrical distance” is associated to each edge of G . For a pair of nodes, let us denote by C_1 and C_2 the two 2×2 sub-matrices of the admittance matrix coupling the pairs of variables associated to these nodes. The weight of the edge between these nodes is defined as

$$\min \left(\frac{1}{\|C_1\|_\infty}, \frac{1}{\|C_2\|_\infty} \right) \quad (6)$$

where

$$\|C\|_\infty = \max_{1 \leq i, j \leq 2} \|C_{ij}\|. \quad (7)$$

- each node at a distance less than a given tolerance tol_{graph} to a fast node is classified as fast.
- all the variables belonging to a fast node or a fast component are classified as fast and will therefore be updated during the refining phase.

NUMERICAL EXPERIMENT

As a numerical experiment we consider the PEGASE problem. This problem is a dedicated test case constructed by the PEGASE consortium [11]. The system modeled is loosely inspired from the European transmission grid in term of size (number of branches, nodes, generators, loads), topology and type of units (nuclear, hydro, TGV). The problem is modeled by a DAE system with 123463 variables, of which 50235 are algebraic.

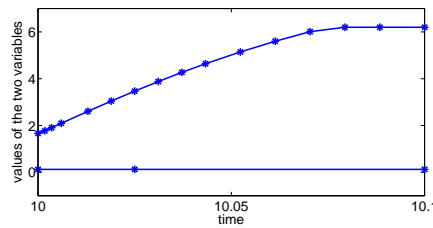


FIGURE 1. Time evolution of one fast and one slow variable.

TABLE 1. Errors and number of function evaluations.

	single-rate	multirate
$\ error\ _\infty$	$4.6 \cdot 10^{-2}$	$5.3 \cdot 10^{-2}$
$\ error\ _2$	$1.3 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$
nr func evals	4938600	1363740
nr Jac evals	2592765	585950
nr Newton iters	4938600	1363740
cost	4.00	0.94

We solve this problem on the time interval $0 < t < T = 10.1$. A short-circuit is performed in the southern Italy during the last 0.1 seconds of simulation time. We expect that this event will only have a local impact and hence, multirate method will be able to exploit this difference in the time scales.

Figure 1 shows the time points in which the solution for two variables, one fast located in Italy and one slow located in Luxembourg, were computed during the time interval when the short-circuit occurred. It is seen that the time steps used for the fast variable are much smaller than the ones used for the slow variable. The solution for the fast variable on this interval is computed by 15 time steps, whereas only 2 time steps are needed for the slow variable.

Table 1 shows the number of function evaluations, number of Jacobian evaluations, number of Newton iterations, estimated costs and the weighted L^2 - and infinity-norm errors (measured with respect to an accurate reference solution) for the single-rate and multirate methods. From these results it is seen that a substantial improvement in cost is obtained. For the single-rate method the estimated costs are four times larger. Moreover, the error behavior of the multirate scheme is very good.

CONCLUSIONS

In this paper we discussed a multirate time stepping approach for the numerical time integration of the European power system model. A strategy for the dynamic partitioning of the components into slow and fast was described. The PEGASE test problem confirmed that the efficiency of time integration methods can be significantly improved by using large time steps for inactive components, without sacrificing accuracy.

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